



Figure 1.: Distribution of costs per load carrier [HB06]

2 THE CURRENT OPTIMIZATION MODEL – A CLOSER LOOK

On pages 161 and 162, [Berbig15] proposes the following optimization model:

$$\min z = \sum_{i=1}^m \sum_{j=1}^n TCO_{ij} x_{ij} \quad (1)$$

$$\text{s. t. } \sum_{i=1}^m \sum_{j=1}^n x_{ij} A_{ij} \leq A \quad (2)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij} l_{ij} \leq L \quad (3)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij} h_{ij} \leq H \quad (4)$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ij} d_{ij} \leq D \quad (5)$$

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i = 1, \dots, m \quad (6)$$

$$x_{ij} \in \{0,1\} \quad \forall i = 1, \dots, m \quad \forall j = 1, \dots, n \quad (7)$$

where:

- TCO_{ij} is the TCO result if load carrier j is used for transporting product i
- A_{ij} is the foot print of bin j containing product i
- l_{ij} is the length of load carrier j transporting product i
- h_{ij} is the according height
- d_{ij} analogously represents the required depth

- x_{ij} means that product i is transported in load carrier j
- A, L, H and D represent total available area, length, height and depth.

The target function (1) aims for the lowest possible costs. Condition (2) guarantees that the required space for load carriers is not exceeding the maximum available area. Conditions (3) to (5) guarantee that length, height and depth e.g. of the shelf or area may not be exceeded. Condition (6) finally states that exactly one load carrier is assigned to one product. This model is working and leads to accordingly valid results. Condition (7) is just signalling that x_{ij} is a binary variable. Now, the question is, if it already considers all aspects that are relevant in practice or if some enhancements are needed.

3 A CLOSER LOOK INTO PRACTICE REQUIREMENTS

The question we have to answer now is: Which requirements have to be fulfilled such that the optimization model does not only work, but is fitting all important production needs? Following Lean principles and according to our experience, the upcoming characteristics have to be given (please note that the order is not linked to the importance of an aspect):

- It must be possible that the material that has been supplied can cover a certain minimum time span at the destination (e.g. to cover at least two milk-run cycles to ensure material is refilled in time)
- Chaotic storing has to be possible e.g. at warehouses
- Kanban (or a Kanban logic) can be used, i.e. at least a 2-bin-system has to be representable
- Following Lean philosophy even further, also supermarkets can be implemented. Thus, single-variety storage is required as well. This means that a certain area is exclusively restricted to a single type of material.
- Besides, practice shows that length, width and depth are considered in an early planning stage already.

Considering these really important requirements as a starting point, what is already considered in the optimization model and what is still missing?

- Covering of minimum amount of time: This is already considered in the TCO-modelling [Berbig15, p. 120]. As the TCO-value is a

part of the target function, this requirement is already considered.

- Chaotic storing: Neither in the TCO-model [Berbig15, p. 91ff] nor in the optimization model (see above) there is a restriction regarding exclusive assignment of storage space or production areas. Thus, this need is satisfied.
- Kanban system: In the current model, it is only guaranteed that a minimum time-span is covered with material. However, it might be that already one bin can fulfil this prerequisite. In this case, only one bin will be considered. Consequently, an adaption is required.
- Supermarket: As mentioned above, there is no restriction of available areas to certain materials. Thus, a supermarket is not considered, yet.
- Explicit consideration of length, depth and height: At least length and depth are already implicitly contained in A. Thus, conditions (3) – (5) might be double considered.

These findings show that the current approach already satisfies important needs, but the optimization model itself can still be extended. How can this be achieved?

4 ENHANCEMENT OF CURRENT OPTIMIZATION MODEL

As seen in chapter 3, enhancements need to be done. This will be done step-by-step in this chapter:

4.1 ENABLING OF A KANBAN-SYSTEM/MULTI-BIN-SYSTEM

Luckily, this requirement is quite easy to fulfil. Basically, only a maximum formula ($\max\{a;b\}$) is needed. The only question is: The maximum of what? Which numbers do a and b represent? The first number, a, is straight forward. It equals the number “m”, according to which kind of m-bin-system is implemented or how many kanbans shall be used. For a 2-bin-system, this would be a 2. B – this is the amount of load carriers required to cover replenishment time – however is slightly harder to define. Let’s consider the following case: One load carrier can cover a time span of 10 min. If the replenishment time is 25 min, we need at least $25 \text{ min}/10 \text{ min} = 2.5$ load carriers. Of course, this number has to be rounded up to a full load carrier, in our case 3. Generalizing this example, one gets:

$$n_{LC,ij} = \max \left\{ m; \left\lceil \frac{RT}{n_{PLC,ij}/DE_i} \right\rceil \right\} \quad (8)$$

where:

- $n_{LC,ij}$ = required number of bins of type j for product i
- m = according type of applied “m”-bin-system or desired number of kanbans
- RT = replenishment time, e.g. [min]
- $n_{PLC,ij}$ = number of pieces per load carrier
- DE_i = Demand of product I per e.g. [min] (according to RT)
- Remaining variables as defined above

4.2 CONSIDERATION OF SUPERMARKETS

Regarding the optimization model, the decisive characteristic of a supermarket which has to be considered is that there have to be dedicated “lines/rows” or areas, where only a certain type of material is allowed. If one bin is within a line of a shelf, this line is restricted to exactly this material. I.e. the whole area this single line covers is now consumed by this one bin already. Consequently, the according question is: How many lines of the supermarket (e.g. shelf) are needed for one material? In order to be able to answer this question, we need to take an assumption. Namely that the load carriers (which are of a rectangular shape) are put into the shelf/area following their longer side. This means they are oriented such that the shorter side is visible from the front of the shelf/area, the longer one is visible from the side. Besides, we assume that the longer side equals the depth of the load carrier, the shorter one is its length. Consequently, we get the following formulae (9) & (10):

$$n_{lines,ij} = \left\lceil \frac{n_{LC,ij}}{\left[\frac{D_{Shelf/Area}}{d_j} \right]} \right\rceil \quad (9)$$

where

- $n_{lines,ij}$ = number of lines needed if product i is in tote j
- $D_{Shelf/Area}$ = available depth of shelf/area
- d_j = depth of load carrier j
- Remaining variables as defined above

Formula (9) also guarantees that every line is exclusively assigned to one product only: The ceil-function applied results in the fact that a whole line is used even if only a fraction of it would theoretically be required.

Having identified the number of lines now, we need to calculate the required area which is a simple area calculation as shown in formula (10):

$$A_{ij} = n_{lines,ij} * D_{Shelf/Area} * l_j \quad (10)$$

where

- A_{ij} = required area if product i is in load carrier j
- l_j = length of load carrier j
- Remaining variables as defined above

4.3 EXPLICIT CONSIDERATION OF LENGTH, DEPTH AND HEIGHT

As already assumed, the explicit consideration of length, depth and height is not required for every single dimension: Formula (9) and (10) include length and depth already for calculation of A_{ij} . Thus, when applying A_{ij} resulting from formula (10) within the optimization model, i.e. ensuring via formula (2) that the available space is not exceeded, it is also guaranteed that both, the available depth and length are not exceeded as well. Thus, the conditions denoted in formulae (3) and (5) can be left aside. But what about formula (4) and the explicit consideration of height? In most cases, also this won't be required: In case of a shelf, the shelf footprint is to be multiplied with the number of levels to calculate the total available space. With doing so, height is considered already and no separate condition is required. If there are further restrictions regarding available height, those should be considered even before starting the TCO analysis, i.e. already when defining the pool of available bins: If a load carrier is too high, it may not be used. Thus, it shouldn't be considered in the available set of load carriers. Consequently, also formula (4) can be neglected. Only in very special cases or to be on the absolute safe side, it can theoretically be considered explicitly. But using it is not recommended.

4.4 THE EXTENDED OPTIMIZATION MODEL

Considering the above findings, we get the following enhanced optimization model (with same basic notations as before):

$$\min z = \sum_{i=1}^m \sum_{j=1}^n TCO_{ij} x_{ij} \quad (11)$$

$$s. t. \sum_{i=1}^m \sum_{j=1}^n x_{ij} A_{ij} \leq A \quad (12)$$

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i = 1, \dots, m \quad (13)$$

$$x_{ij} \in \{0,1\} \quad \forall i = 1, \dots, m \quad \forall j = 1, \dots, n \quad (14)$$

where

- A_{ij} = required area calculated according to (10)
- Remaining variables as defined above

This model has less boundary conditions, as the former equations (3) – (5) are already considered within calculation of A_{ij} and can consequently be neglected. Even if formula (11) and (12) look like before, there is an important difference: A_{ij} is now different, as it is now calculated according to formula (10). The remaining formula and variables remain unchanged.

5 APPLYING THE ENHANCED OPTIMIZATION MODEL

Having defined the enhanced optimization model, it is now important to check if it really works. Thus, we consider an illustrative example: Let's assume we have 10 different products and 14 different load carriers. These load carriers have the following dimensions where SLC represents the group of "small" load carriers and LLC the one of "large" load carriers:

Load Carrier Footprint		LC dimensions [cm]
0,06	SLC	20*30
0,06		20*30
0,12		30*40
0,12		30*40
0,24		40*60
0,48	LLC	60*80
0,48		60*80
0,48		60*80
0,48		60*80
0,96		80*120
0,96		80*120
0,96		80*120
0,96		80*120
0,96		80*120

Figure 2.: Dimensions of exemplary load carriers

With these, we assume to have calculated the following results for A_{ij} if product i is transported within bin j when applying formula (10), see **Figure 3**.

Load carrier	Aij for product... in load carrier...									
	1	2	3	4	5	6	7	8	9	10
1	0,24	0,3	0,48	1,2	0,24	0,3	0,48	1,2	0,48	0,12
2	0,24	0,24	0,36	1,08	0,24	0,24	0,36	1,08	0,36	0,12
3	0,36	0,36	0,48	1,08	0,36	0,36	0,48	1,08	0,48	0,24
4	0,36	0,36	0,36	0,96	0,36	0,36	0,36	0,96	0,36	0,24
5	0,48	0,72	0,72	1,2	0,48	0,72	0,72	1,2	0,72	0,48
6	0,96	1,44	1,44	2,4	0,96	1,44	1,44	2,4	1,44	0,96
7	0,96	1,44	1,44	1,92	0,96	1,44	1,44	1,92	1,44	0,96
8	0,96	0,96	1,44	0,96	0,96	0,96	1,44	0,96	1,44	0,96
9	0,48	0,96	1,44	0,96	0,48	0,96	1,44	0,96	1,44	0,96
10	0,96	0,96	1,92	1,92	0,96	0,96	1,92	1,92	1,92	1,92
11	0,96	0,96	1,92	1,92	0,96	0,96	1,92	1,92	1,92	1,92
12	0,96	0,96	1,92	1,92	0,96	0,96	1,92	1,92	1,92	1,92
13	0,96	0,96	1,92	1,92	0,96	0,96	1,92	1,92	1,92	1,92
14	0,96	0,96	1,92	1,92	0,96	0,96	1,92	1,92	1,92	1,92

Figure 3.: Exemplary A_{ij}

Load carrier	Costs for product... in load carrier...									
	1	2	3	4	5	6	7	8	9	10
1	5.912,00 €	5.667,00 €	5.025,00 €	5.280,00 €	5.712,00 €	5.219,00 €	5.196,00 €	5.362,00 €	5.359,00 €	5.228,00 €
2	5.737,00 €	5.872,00 €	5.753,00 €	5.293,00 €	5.266,00 €	5.925,00 €	5.742,00 €	5.997,00 €	5.917,00 €	5.832,00 €
3	5.857,00 €	5.594,00 €	5.847,00 €	5.279,00 €	5.856,00 €	5.597,00 €	5.950,00 €	5.263,00 €	5.824,00 €	5.413,00 €
4	5.691,00 €	5.580,00 €	5.719,00 €	5.162,00 €	5.139,00 €	5.899,00 €	5.846,00 €	5.437,00 €	5.675,00 €	5.794,00 €
5	5.355,00 €	5.180,00 €	5.075,00 €	5.966,00 €	5.448,00 €	5.179,00 €	5.845,00 €	5.252,00 €	5.489,00 €	5.125,00 €
6	5.643,00 €	5.904,00 €	5.090,00 €	5.024,00 €	5.661,00 €	5.370,00 €	5.898,00 €	5.801,00 €	5.302,00 €	5.031,00 €
7	5.198,00 €	5.156,00 €	5.802,00 €	5.148,00 €	5.416,00 €	5.057,00 €	5.312,00 €	5.087,00 €	5.599,00 €	5.541,00 €
8	5.728,00 €	5.055,00 €	5.381,00 €	5.634,00 €	5.880,00 €	5.897,00 €	5.254,00 €	5.192,00 €	5.346,00 €	5.295,00 €
9	5.942,00 €	5.724,00 €	5.696,00 €	5.437,00 €	5.664,00 €	5.266,00 €	5.454,00 €	5.305,00 €	5.245,00 €	5.249,00 €
10	5.505,00 €	5.816,00 €	5.747,00 €	5.181,00 €	5.382,00 €	5.300,00 €	5.184,00 €	5.502,00 €	5.796,00 €	5.353,00 €
11	5.171,00 €	5.840,00 €	5.978,00 €	5.130,00 €	5.862,00 €	5.669,00 €	5.891,00 €	5.345,00 €	5.802,00 €	5.272,00 €
12	5.612,00 €	5.346,00 €	5.286,00 €	5.888,00 €	5.569,00 €	5.803,00 €	5.612,00 €	5.731,00 €	5.825,00 €	5.093,00 €
13	5.012,00 €	5.656,00 €	5.344,00 €	5.540,00 €	5.511,00 €	5.106,00 €	5.939,00 €	5.764,00 €	5.791,00 €	5.771,00 €
14	5.507,00 €	5.379,00 €	5.118,00 €	5.516,00 €	5.617,00 €	5.023,00 €	5.160,00 €	5.265,00 €	5.948,00 €	5.384,00 €
TCO for product i	5.012,00 €	5.055,00 €	5.025,00 €	5.024,00 €	5.139,00 €	5.023,00 €	5.160,00 €	5.087,00 €	5.245,00 €	5.031,00 €

Figure 4.: TCO for all load carrier and product combinations

Load carrier	Costs for product... in load carrier...									
	1	2	3	4	5	6	7	8	9	10
1	5.912,00 €	5.667,00 €	5.025,00 €	5.280,00 €	5.712,00 €	5.219,00 €	5.196,00 €	5.362,00 €	5.359,00 €	5.228,00 €
2	5.737,00 €	5.872,00 €	5.753,00 €	5.293,00 €	5.266,00 €	5.925,00 €	5.742,00 €	5.997,00 €	5.917,00 €	5.832,00 €
3	5.857,00 €	5.594,00 €	5.847,00 €	5.279,00 €	5.856,00 €	5.597,00 €	5.950,00 €	5.263,00 €	5.824,00 €	5.413,00 €
4	5.691,00 €	5.580,00 €	5.719,00 €	5.162,00 €	5.139,00 €	5.899,00 €	5.846,00 €	5.437,00 €	5.675,00 €	5.794,00 €
5	5.355,00 €	5.180,00 €	5.075,00 €	5.966,00 €	5.448,00 €	5.179,00 €	5.845,00 €	5.252,00 €	5.489,00 €	5.125,00 €
6	5.643,00 €	5.904,00 €	5.090,00 €	5.024,00 €	5.661,00 €	5.370,00 €	5.898,00 €	5.801,00 €	5.302,00 €	5.031,00 €
7	5.198,00 €	5.156,00 €	5.802,00 €	5.148,00 €	5.416,00 €	5.057,00 €	5.312,00 €	5.087,00 €	5.599,00 €	5.541,00 €
8	5.728,00 €	5.055,00 €	5.381,00 €	5.634,00 €	5.880,00 €	5.897,00 €	5.254,00 €	5.192,00 €	5.346,00 €	5.295,00 €
9	5.942,00 €	5.724,00 €	5.696,00 €	5.437,00 €	5.664,00 €	5.266,00 €	5.454,00 €	5.305,00 €	5.245,00 €	5.249,00 €
10	5.505,00 €	5.816,00 €	5.747,00 €	5.181,00 €	5.382,00 €	5.300,00 €	5.184,00 €	5.502,00 €	5.796,00 €	5.353,00 €
11	5.171,00 €	5.840,00 €	5.978,00 €	5.130,00 €	5.862,00 €	5.669,00 €	5.891,00 €	5.345,00 €	5.802,00 €	5.272,00 €
12	5.612,00 €	5.346,00 €	5.286,00 €	5.888,00 €	5.569,00 €	5.803,00 €	5.612,00 €	5.731,00 €	5.825,00 €	5.093,00 €
13	5.012,00 €	5.656,00 €	5.344,00 €	5.540,00 €	5.511,00 €	5.106,00 €	5.939,00 €	5.764,00 €	5.791,00 €	5.771,00 €
14	5.507,00 €	5.379,00 €	5.118,00 €	5.516,00 €	5.617,00 €	5.023,00 €	5.160,00 €	5.265,00 €	5.948,00 €	5.384,00 €
TCO for product i	5.012,00 €	5.055,00 €	5.025,00 €	5.024,00 €	5.139,00 €	5.023,00 €	5.160,00 €	5.087,00 €	5.245,00 €	5.031,00 €

Figure 5.: Results of the enhanced optimization model

Taking these values into consideration, we assume to have calculated TCO as shown in **Figure 4**.

The cells highlighted in light blue do always show the TCO optimal load carrier for each product. In this case, the overall cost would be **50,801.00 €**.

With this information, we are now able to solve the optimization model with a simple linear solver even within Microsoft Excel. In order to do so, we simply implement the enhanced optimization model into the solver. We get the results as shown in **Figure 5** where the dark blue cells are the resulting cells calculated from the solver, the light blue ones are again the TCO optimal ones.

The first interesting and important finding is that the TCO optimal load carrier is not necessarily also the overall optimum one. Besides, now the overall cost is **52,242.00 €**.

This means, overall cost has increased by 1,441 € or approx. **2,84 %**. Why is this the case? One answer is that there might be synergy effects when having only a limited range of different load carriers. However, this is not explicitly modelled, yet. The answer is that there is another decisive boundary: The total available area. In the above-mentioned case, the available area is only 5 m². Using the TCO optimal load carriers only, **12.36 m²** would be required which are not available. Consequently, this is not a possible solution. With the optimization model, the result leads to **4.92 m²**. This shows that the enhanced optimization is able to satisfy all boundary conditions and practice requirements mentioned above. Thus, it is suitable and can be applied.

6 SUMMARY

Cost saving via optimized processes is crucial. However, not only optimized processes offer according potential, but also the tool with which a process is performed. Tools like the used load carriers. Even if there is a sheer endless amount of standardized load carriers and bin management has become broadly applied within the last years and decades, important factors are still neglected and load carriers are not considered comprehensively. Neither in practice nor in theory. There are only a few publications on this topic out of which one has been taken as basis for this research. This previous work presents a TCO modelling approach and afterwards a first optimization model. In this paper, we have a close look into practice requirements. The performed cross-check shows that enhancements to the optimization model are supportive. Consequently, an enhanced model has been developed taking these requirements into consideration. This model proves to be efficiently working and considers the identified practical needs. As it is a linear one, it can easily be implemented and solved with standard tools and solvers. However, some questions still remain open offering room for further

research like allowing only a certain number of different load carriers or repacking at production. Nevertheless, first important enhancements have been made, an enhanced model suiting practice needs is presented. With the previous optimization model and the enhancements shown in this paper, it is now possible to consider these highly important parameters within the optimization of the set of load carriers used:

- Kanban systematics
- Chaotic storing (A_{ij} to be calculated as derived in [Berbig15])
- Supermarket (A_{ij} to be calculated as developed in this paper)
- Guaranteeing of a minimum timespan that is covered by material
- Consideration of restrictions at destination (e.g. available area).

Fulfilling these characteristics, the model is applicable for many use cases. Nevertheless, there remain still some open questions, e.g. like restricting the model to a maximum number of allowed load carriers. These have to be checked in further research works.

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