

Queueing Models for manual order picking systems with blocking

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In many picker-to-part order picking systems with high space utilization, blocking situations occur and lead to productivity losses. On the one hand order pickers cannot pass each other within aisles and cross-aisles, on the other hand they want to access a certain resource of the system, for example a base station, at the same time. Very few papers and articles dealing with this problem are available in the literature and the few approaches only offer limited possibilities of modeling and quantifying. For this reason the Deutsche Forschungsgemeinschaft (DFG) is funding a research project which aims at developing theoretical foundations to understand blocking in manual order picking systems and quantify its effects by means of an analytical model. In this paper we identify queueing theory as a potential modeling method. We show how to transfer a manual order picking system into a queueing model and present applicable solution algorithms. The analysis of an exemplary system shows that blocking situations reduce the productivity of an order picker and that throughput losses can be as high as 10% in typical implementations. For these scenarios we were able to show that the relative error between the queueing model and simulation is below 4%. Therefore queueing theory is suited as a modeling method because compared to simulation, results can be derived fairly easy and in short time.

1. Introduction

Order picking systems are an important element of facility logistics. Even though the possibilities of organizing such systems seem to be unlimited, most warehouses use Picker-to-parts configurations. Data from real distribution centers collected in the Warehouse Excellence study show that 80% of costs can be attributed to such manual systems (DCRM 2008).

In manual order picking systems where items are statically stored in racks, workers beside their productive work time, spend a certain amount of time waiting. The unbalanced utilization of different picking areas may be one reason for this. Another reason, which hasn't gained much attention neither in real applications nor in the scientific community, is blocking. Blocking occurs at the base station or within aisles when an order picker is interrupted or interfered while processing his or her order. Because of narrow aisles, passing other order pickers is often hindered or impossible. The blocking situations result in higher throughput times and thus lower picker productivity and lower overall order throughput. Narrow aisles are the direct result of a high space utilization which is desired in order to cope with increasing product lines and rising energy costs.

The literature review shows that few papers dealing with order picking systems have considered blocking. Instead most authors use static approaches to analyze performance in manual order picking systems. In the basic work in this field, Kunder and Gudehus (Kun-75) derive expected travel times of an order picker using static models. Based on this, Hall (Hall-93) developed approximations for path lengths. To optimize order picker travel, Ratliff and Rosenthal (Rat-83) transform the layouts of different order picking systems into graphs and solve the problem using a modified Travelling-Salesman-Problem (TSP). Roodbergen and de Koster (Rood-01) combined the TSP with a sensitivity analysis to include shortcuts of order pickers.

Static approaches are mostly used when planning and organizing order picking systems. The results of these models can be translated into a standardized key performance indicator, such as the number of completed customer orders per hours or number of picked order lines per man-hour. Based on the performance of one order picker it is suggested that the output of multiple order pickers continue to rise straightly proportional. The problem with this planning procedure is shown in figure 1. With an increasing number of order pickers in the system there will be a rising gap between the estimated throughput and the assumed trend which includes dynamic effects caused by blocking.

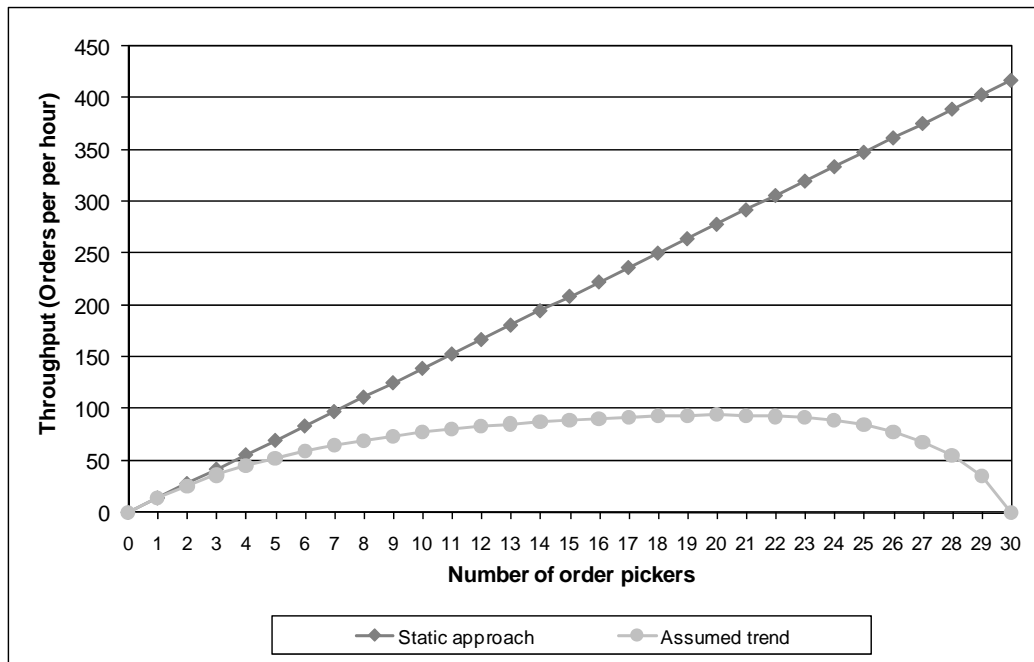


Figure 1: Relation between number of order pickers and order throughput

A first consideration of the possible impacts of blocking is given in Ruben and Jacobs (Rub-99), Faißt and Lippolt (Fai-02) and Arnold (Arn-03). They question the linearity of the throughput trend and notice that order pickers interfere with each other. In his dissertation Lünig (Lün-05) studies declining throughput trends caused by rising number of order pickers in more detail. He differentiates in blocking caused by two order pickers trying to pick from one rack column at the same time and interferences when order pickers meet in the aisles. Using probability theory he derives a percental throughput loss rate. The loss rate can be applied to the results of static models to obtain the real system throughput. In his work Lünig examines only isolated aisles and therefore routing strategies were not analyzed. Also when pickers meet in the aisles their speed is assumed to decrease but no formation of queues is considered. Another work seeking to estimate the blocking rate was delivered by Gue, Meller and Skufca (Gue-06) in their congestion model. They transformed the routes of order pickers into a closed loop without passing. On the basis of a fixed ratio between picking and walking time as well as the probability of a pick, the authors derive a blocking rate which indicates the percentage of time a worker is blocked. Due to the model assumption, only few forms of organization can be analyzed regarding blocking. Furthermore the chosen ratios do not represent observations in real order picking systems.

Based on the lack of research considering blocking phenomena, the research project “Blocking in manual order picking systems with Picker-to-Part movements” is funded by the Deutsche Forschungsgemeinschaft. The project aims at developing theoretical foundations to specify and quantify blocking phenomena in order to obtain a better understanding for the resulting decrease in productivity. Furthermore strategies for an efficient order picking process with consideration of blocking are developed.

This paper at first characterizes different basic forms of blocking. In order to use a queueing model for calculation of the effects of blocking, it is shown how the parameters of an order picking system can be transferred. Subsequently solution algorithms and their applicability are discussed. The results of the queueing model are compared to simulation in order to legitimate the use of queueing theory.

2. Characteristics of blocking situations

The quantification of reduced output caused by blocking in manual order picking systems requires a definition of situations to be considered blocking situations. Blocking basically results in waiting times. Waiting times can occur without blocking though, for example if order pickers have no new orders to process. Blocking situations can be distinguished from such waiting situations using the following definition:

Definition: A situation is called blocking situation, whenever the activities of an order picker are interrupted by another order picker.

In manual order picker systems a blocking situation arises whenever two or more pickers want to use one of the following resources at the same time:

- A rack column within the aisle and the space in front of the rack column
- A space of the cross-aisles
- The base station and the related space in front of the base station

2.1. Blocking situations in aisles and cross-aisles

The aisles and cross-aisles of a manual order picking system consist of a certain number of elementary spaces. An elementary space is a sector of an aisle or cross-aisle, for example the space in front of two opposite rack columns. The elementary space can be used by only one order picker at a time if the width of the aisles falls below a certain minimum measurement. A blocking situation will occur if the elementary space is occupied by one order picker and another order picker needs the space to either pick from the rack column or pass on his way through the aisle. Subject to the direction of the order picker movement blocking situation can be either simple or more complex (see figure 2).

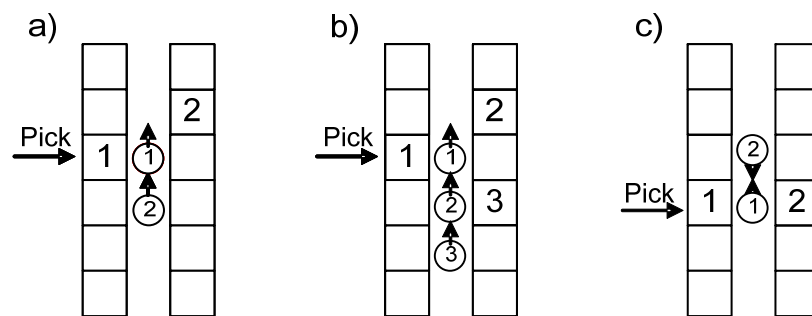


Figure 2: Exemplary blocking situations

We assume the direction of movement to be fix in examples a) and b) and to be arbitrary in example c).

If order picker 1 picks his next orderline from rack column 1 in example a) and therefore uses the related elementary space, a level-one blocking situation can occur when order picker 2 wants to pass in order to reach rack column 2. Order picker 2 has to wait until order picker 1 has fully finished his orderline and continues walking the aisle to his next stop.

Example b) partly shows the same situation as example a). While order picker 2 is blocked by order picker 1, he occupies the elementary space in front of rack column 3. As order picker 3 traverses the aisle, he is blocked by order picker 2. This represents the case in which an order picker who is already blocked causes a blocking situation by himself. We call this situation a level-two blocking situation. Let k be the number of order pickers. Then basically a level- $(k-1)$ blocking situation can result and the blocking queue for this case will be $(k-1)$ order pickers. Because the direction of order picker movement is strictly one-way in this configuration, a reduction in the queue length will result from a first-come-first-serve processing rule. As in the case of a simple queueing system the last arriving order picker has to wait until all order pickers in front of him have been served.

Blocking situations resulting from missing one-way-traffic rules are presented in example c). Order picker 1 is picking from rack column 1, thus occupies the associated elementary space and after finishing the pick has to walk to the top according to the direction of the arrow. Order picker 2 needs to pick from the opposite rack column 2 and is blocked by order picker 1. Consequently a level-one blocking situation exists (1 is blocking 2). As soon as order picker 1 has finished picking, he is blocked by order picker 2 because of the different directions of both order pickers. (2 is blocking 1) The termination of this level-two opposite blocking situation is more complicated than examples a) and b). To avoid deadlocks, the situation requires definite priority rules which determine the following walking process by regulating which of the two order pickers has to exit the aisle by changing his original direction.

2.2. Blocking situations at the base station

At the base station of a manual order picking system the order pickers receive the picking list which can represent a single customer order or could be a combination of several orders. A picking list consists of order

lines, which specify the items, its storage locations and the quantities. Besides the order pickers receive empty bins in which picked items are placed. When all order lines of an order are picked, the order picker returns to the base station. Like in aisles and cross-aisles an order picker uses the base station and the associated elementary space as a resource for a certain period of time. If in the same space of time a second or a k -th order picker needs the same resource the order picker or all $(k-1)$ order pickers respectively have to wait. The complexity of such blocking situations is bounded and the blocking will dissolve by applying simple FIFO-logic.

3. Parameters of a manual order picking system

The layout of manual order picking systems is determined by the use of several resources. Multiple aisles, characterized by a certain number of elementary spaces and associated opposite rack columns, are connected by cross-aisles. Within the aisles the workers pick the respective items from the rack columns. The typical block layout is completed by a base station and the manual order picking system represents a closed area, where workers start and end their tours at the base station (see figure 3). The number of aisles and rack columns within the aisles, as well as the number and location of base stations represent the structural parameters of the system.

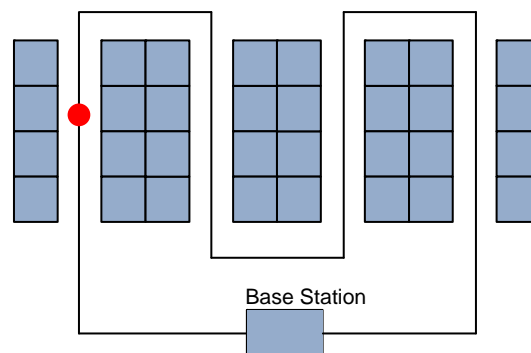


Figure 3: Exemplary order picking system with a picker-to-part configuration

Apart from the physical configuration an order picking system is characterized by the organization of the order picking process itself. The number of order pickers is an output-determining parameter and is a significant driver to manage the number of completed customer orders. In addition, the performance of the system depends on the time, which is spent at the respective resources. One has to consider the time needed at the base station, the dead and picking times needed at the rack columns and the travel times to walk through the aisles and cross-aisles. The base time is determined by the design and organization of the base station. Dead times can be reduced by tools supporting the order picker, for example while searching an orderline. The picking times depend on the order-specific quantities as well as the weight and volume of items. The travel times result from the speed of order pickers and their walking routes, which are set by the particular routing strategies. These define the possible movements of order pickers and thus the potential blocking situations, which can occur in the system (see figure 2). Order-specific parameters like the number of order lines per order or item-specific parameters like the allocation of items to the aisles also influence the routing of order pickers.

4. Queueing model of a manual order picking system

Order picking systems are affected by numerous stochastic effects. Considering a single day or shift, customer orders usually don't enter the system uniformly distributed, but can be characterized by peaks. Therefore not only the mean value of arriving orders but also the standard deviation has to be considered. The same applies to service times, the size of an order etc. On the basis of certain input parameters queueing theory can calculate characteristic values like the average throughput time or the average length of a queue. Queueing theory is used in a wide range of applications and has the edge over static methods because input parameters are statistical values and so allows for stochastic effects to be considered. A detailed discussion about the fundamentals of queueing theory is given for instance by Bolch (Bol-98).

To model and evaluate a manual order picking system using queueing theory the parameters of the system (number of order pickers, resources, times and routings) have to be transferred into the input parameters of the queueing model (number of customers k , number of elementary queueing systems n , capacities of each elementary queueing system m_i , service times μ_i , variabilities c_i^2 and transition probabilities q_{ij}).

4.1. Transformation of a manual order picking system into a queueing network

4.1.1. Modeling the number of order pickers and choosing the type of network

In the queueing model the number of order pickers is represented by the number of customers k in the system. In real systems the number of order pickers is fix over a certain period of time. It can rise in peak hours but then stays on this elevated level for a while.

Based on the assumption of a fixed number of customers a *closed queueing network* matches the real system best. The resulting throughputs λ (= completed customer orders) are variable and state-dependent. For example if all k order pickers reside in one elementary queueing system i , the state-dependent throughput rate of this system $\lambda_i(k) = 0$. Using a closed queueing network also implies that the base station always has enough customer orders ready for processing.

Open queueing networks are considered to be less applicable because the throughput value λ corresponds to the number of finished customer orders and is constant in such networks. However the number of order pickers (= number of customers in the system) is variable. This would not comply with the assumption that the number of order pickers is fix.

4.1.2. Modeling resources of an order picking system

The transformation of the resources is done based on the logic of the elementary spaces. Each potential location of an order picker (space in front of a rack column, space of a cross-aisle, space in front of the base station) is defined as an elementary space which can be occupied by only one order picker at a time. Each elementary space can then be modeled by an elementary queueing system. An order picker (=customer) enters the elementary space (=queueing system) and has to either pass it or depending on the type of resource make a pick or deliver a completed order at the base station and then enters the successive elementary space. The sum over all elementary spaces corresponds to the number of queueing systems n . Linking all queueing systems results in a closed queueing network which represents the order picking system.

Modeling the rack columns

Within an aisle the elementary space represents the space in front of two opposite rack columns. *Each elementary space is a queueing system with one server and a buffer of size 0. An aisle is made up by a sequence of several such queueing systems.* The assumption that buffers are non-existent (buffer size = 0) avoids the overlapping of a buffer with the server of a preceding queueing system. Let a certain rack column and the corresponding elementary space in front of it be represented by a queueing system n , then n itself is the buffer (with capacity 1) of the succeeding queueing system ($n+1$). Respectively the preceding queueing system ($n-1$) is the buffer (with capacity 1) of queueing system n .

Modeling the cross-aisles

Depending on the length of the cross-aisle it can hold either one or multiple order pickers. The cross-aisle can be modeled as an elementary space or a sequence of elementary spaces, with each elementary space being represented by an elementary queueing system (buffer size = 0).

Modeling the base station

The base station as well as the routes leading to and from the base station can also be considered as elementary spaces and thus be modeled as elementary queueing systems. Each elementary space represents the waiting room of the succeeding elementary space. Therefore all queueing systems have one server and no buffers.

The example of the order picking system shown in figure 3 can be modeled as a closed queueing network with several elementary queueing systems (each with capacity $m_i = 1$) as follows:

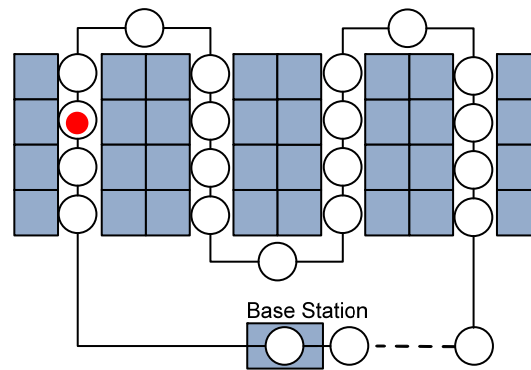


Figure 4: Order picking system modeled as a closed queueing network

4.1.3. Modeling times

Depending on the type of resource, order pickers have to perform different activities at the elementary spaces. The time needed for these activities has to be included in the service times of the corresponding queueing systems. The **travel time** must always be a part of the service time because each queueing system represents an elementary space and thus a certain distance to be travelled. Assuming constant speed of the order pickers, the travel time can then be modeled as a deterministic variable. In the cross-aisles there are no other activities than walking and therefore the service time equals the travel time. Whenever a queueing system represents a base station the **base time** for handing over completed and receiving new customer orders is added to the deterministic travel time to obtain the overall service time. The number of order lines and the location of the particular items determine the probability of an order picker to stop and pick at a certain rack column. There may be rack columns which have a higher picking probability because the items stored in these rack columns have a higher turnover rate than other items. Thus with a certain probability the service time of a queueing system additionally has to consider the **dead and picking time**. The picking time increases depending on item-specific characteristics (weight, volume) and the picking quantity.

In real systems, the base, dead and picking times are random variables. It has to be determined, by which statistical distributions they can be represented. Many methods for calculating performance measures of queueing models assume exponentially distributed service times. For order picking systems this assumption is rather problematic because each activity requires a certain minimum time. For exponential distributions realizations close to zero have the highest probability and thus a realistic modeling of actual processing times is not given. A better alternative for modeling the base, dead and picking times is the logarithmic normal distribution (Kre-08). The random variable X is lognormal distributed, if $\ln(X)$ is normally distributed. A lognormal distributed random variable has positive values. Its right-skewness reflects the real observation that the mean value of the processing times lies to the right of the mode of the distribution. In reality base, dead and picking times which are bigger than the mode are completely feasible. Times lower than the mode are rarely observed. Lognormal distributed times can be characterized by the mean value and the standard deviation: $X \sim \text{LogN}(\text{Mean Value}, \text{Standard Deviation})$.

Because the base, dead, picking and travel times are independent random variables and deterministic variables respectively, their mean values and variances can be added up. Because of the assumption that the base, dead and picking times are lognormal distributed, the service process of the queueing systems can be represented by a general distribution with mean service rate μ_i and variability c_i^2 . The variability is a measure for process variations and is calculated from the ratio of the variance of the service time and its squared expected value.

4.1.4. Modeling the routing

The routing strategies of order pickers determine how they walk through the system, i.e. in which succession they have to visit the resources (rack columns, cross-aisles, base station). According to Gudehus (Gud-05) the following routing strategies are of significant importance in manual order picking systems where items are statically stored in racks:

- Traversal (S-shape) strategy without aisle skipping
- Traversal (S-shape) strategy with aisle skipping and one-way traffic
- Return strategy with single aisle visits / multiple aisle visits

The routing in closed queueing networks is determined by the transition probabilities. In a queueing model the transition probabilities specify how the elementary queueing systems are linked with each other. The transition

probability q_{ij} indicates the probability that a customer will have to visit queueing system j after having finished service at queueing system i .

Traversal (S-shape) strategy without aisle skipping

With this strategy in use, order pickers traverse through all aisles of the system in a pre-defined succession (see figure 4), irrespective of the fact that certain aisles may be traversed even though no pick is performed in these aisles. The corresponding queueing model has a strictly serial arrangement, i.e. each queueing system i has exactly one successor j and $q_{ij} = 1$.

Traversal (S-shape) strategy with aisle skipping and one-way traffic

In this strategy aisles may be skipped if there are no picks in the aisles. The one-way traffic rules prevent opposing routes. Aisles with an even number have to be traversed in a designated direction. In aisles with an odd number only movements in the opposing direction are permitted. Thus complex blocking situations in which pickers meet each other face to face (figure 2, c) within an aisle are impossible.

To transfer this strategy, branching elements are added at certain points in the closed queueing network. At these elements order pickers have to decide which route to continue. The branching elements are located in front of the aisle entrances and behind the aisle exits respectively. Figure 5 shows the queueing model of this routing strategy.

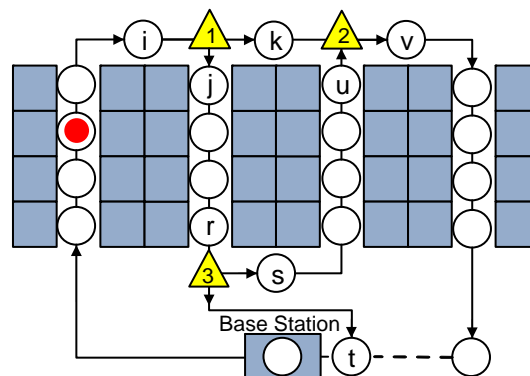


Figure 5: Order picking system modeled as a closed queueing network – Traversal (S-shape) strategy with aisle skipping and one-way traffic

After passing through the upper left cross-aisle (queueing system i) the order picker has to visit queueing system j next, if he has to pick an item in aisles number two or three or has to get back to the base station (due to one-way traffic rules the order picker has to traverse aisle two even if his next pick is located in aisle three). If the next pick is located in aisle number four, the picker can move straight on to queueing system k and from there on to queueing system v . Therefore the transition probability q_{ij} depends on the probability that a pick is located in aisle two or three and it is required that $q_{ij} \in [0;1]$ as well as $q_{ij} + q_{ik} = 1$. The same holds for the third branching element, where q_{rs} corresponds to the probability that the order picker has at least one pick in aisle three or four. q_{rt} indicates the probability that an order picker has no more picks on his current route and has to return to the base station. Within the aisles each queueing system has only one successor. By defining certain transition probabilities to be zero, i.e. $q_{uk} = 0$, we guarantee the adherence to one-way traffic rules.

The value of the transition probabilities ultimately depends on the picking probability of each item. If fast-moving items are stored exclusively in aisles one and two while slow-moving items are stored in aisles three and four, then q_{ij} will be larger than q_{ik} .

Return strategy with single aisle visits / multiple aisle visits

In this strategy order pickers move along a main aisle which runs upright to the picking aisles. Whenever an item has to be picked from a certain aisle, the order picker leaves the main aisle. If the order picker travels back to the main aisle after a pick – for example to place items on a picking trolley – and reenters the aisle for another pick, the strategy includes multiple aisle visits. If the order picker enters an aisle, carries out all picks in this aisle and finally returns to the main aisle, the strategy includes single aisle visits. Whenever an order picker enters an aisle and the whole aisle is subsequently blocked for other order pickers, an exclusive aisle reservation is in effect. Alternatively if multiple order pickers share one aisle simultaneously, the aisle reservation is non-exclusive. The first case is shown in figure 6.

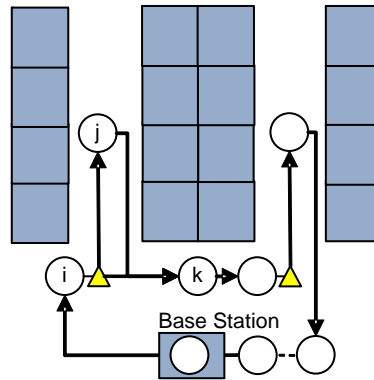


Figure 6: Order picking system modeled as a closed queueing network – return strategy

In contrast to the previous approach, not each elementary space within an aisle is modeled as an elementary queueing system. In fact the whole aisle is considered to be an elementary space so the whole aisle is represented by one queueing system which has no buffer. As soon as one order picker enters the aisle, all other order pickers have to wait in front of the aisle (exclusive aisle reservation). The transition probability q_{ij} indicates the probability that an aisle (represented by queueing system j) has to be entered. The service time of j includes all dead, picking and travel times of an order picker. After exiting queueing system j the order picker moves back onto the main aisle, which is a succession of elementary queueing systems. If the main aisle has sufficient width available, the queue in front of an aisle does not impact movements on the main aisle. It can then be modeled by two parallel queueing systems. One queueing system represents the entrance to the aisle (which will hold potential queues), while the second queueing system represents a sector of the main aisle. This approach can be used for both single or multiple aisle visits. The latter, based on the additional travel times, results in a longer service time of the queueing system representing the aisle. In the case of non-exclusive aisle reservation the problem of opposing traffic may occur.

In closed queueing networks the transition probabilities are transformed into so-called visit ratios. The visit ratio e_i indicates the relative share of the overall flow, which flows through queueing system i . The base station acts as counting node because it has to be visited once on each route.

Figure 7 summarizes the transfer of parameters of a manual order picking system into the input parameters of a queueing model.

Parameters of the order picking system	Queueing Model
Number of order pickers	Number of customers k
Rack columns, cross-aisles, base station	Number of queueing systems n Capacity of the queueing system m_i
Base, dead, picking and travel times	Service rate μ_i , variability c_i^2
Order-specific data (# order lines, etc.)	Service rate μ_i , variability c_i^2 , visit ratios e_i
Routing strategy	Visit ratios e_i
Item-specific data (allocation, etc.)	Visit ratios e_i

Figure 7: Parameters of an order picking system and their correspondent parameters in a queueing model

4.2. Computation of characteristic values for the queueing model

The transfer of a manual order picking system into a queueing model is based on the following assumptions:

- Closed queueing network with n queueing systems and k customers
- Either cyclic or arbitrary configuration of the elementary queueing systems
- Capacities of the queueing systems m_i are 1 respectively, i.e. the size of the buffer is zero
- Service times have a general distribution (mean service rate μ_i and variability c_i^2)

Due to these assumptions, numerous calculation techniques used in queueing theory are not applicable. Especially the assumptions that service times are not exponentially distributed as well as the finite capacities of buffers reduce the set of potential calculation methods.

The methods of Akyildiz developed in 1987 (Aky-87), Bouhchouch et al. (Bou-96) and Rall (Rall-98) are potential methods for the analysis. In his publication (Aky-87), Akyildiz shows relatively large errors of 20-25%. The approach of Bouhchouch et al. can be classified as a decomposition method and therefore tends to be computationally complex for larger networks. Based on the accurate results, which were reported for networks with cyclic configurations, the method of Rall seems to be a suitable approach.

The method of Rall combines well-established solution algorithms into a new calculation framework. First the method uses the approach of Akyildiz developed in 1988 (Aky-88). Akyildiz transforms a blocking network (queueing network with finite capacity buffers) with exponentially distributed service times and k customers into an equivalent non-blocking network with a new number of customers k^* , with $k^* \leq k$. Due to the assumption of exponentially distributed service times, the approach of Akyildiz disregards the effects of variabilities $c_i^2 \neq 1$.

Rall adjusts the reduced number of customers k^* with the help of a correction factor f to get $k^{**} = k^* \cdot f$. The correction factor f depends, among other things, on the variability of the elementary queueing systems. Similarly the ratio of customers currently in the system to the maximum possible number of customers in the system influences the value of the correction factor. To calculate f , Rall uses the mean value analysis, an important method to calculate throughput with unrestricted buffer size and exponentially distributed service times (Bol-98).

In the last step, the method of Rall utilizes the method of Marie (Mar-80), using k^{**} as the number of customers. First of all, the iterative method calculates the arrival rates for the corresponding number of customers for each queueing system. Depending on the variability the service process is modeled by k -phase Coxian distributions and the service rates are determined subject to the number of customers. Marie's method originally calculates performance indicators for closed queueing networks with unrestricted buffer size and general service times. By using the adjusted number of customers k^{**} , Rall's method allows for the consideration of effects caused by finite buffers.

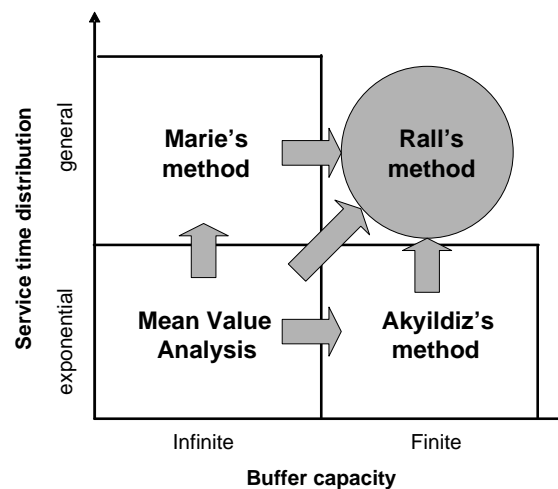


Figure 8: Selected solution algorithms for closed queueing networks

5. Exemplary calculation of performance indicators for a manual order picking system

5.1. Modeling of an example of an order picking system

The calculation of performance indicators is carried out in order to evaluate the quality of the results of the presented queueing model. The first step of this includes the design of a typical manual order picking system. Based on this, performance indicators calculated by usage of queueing theory and simulation are compared and relative errors derived.

We assume the exemplary system to have one base station. The system with one block has four aisles, each with 15 elementary spaces (= 30 rack columns). The length of an aisle is 15m. The depth of a rack column is 0,75m, the overall length of a cross-aisle sector, connecting two aisles, is 3m. The distance between the last aisle and the base station is 5m. The cross-aisles and the base station are represented by one single elementary space each while the distance from the last aisle to the base station is made up of five elementary spaces. From this it follows that five order pickers can wait in front of the base station without initiating blocking situations within the aisles. The overall system consists of 69 elementary spaces. Each picking order consists of 10 order lines, each with quantity 1. The picking probability is the same for all rack columns. The base station always holds a sufficient number of new customer orders, so an order picker never has to wait for a new order. The speed of an order picker is fix 1 m/s, acceleration is disregarded. The routing strategy is the traversal (S-shape) strategy without aisle skipping. Dead, picking and base times are lognormal distributed and vary in the different scenarios. For all scenarios we assume that dead and picking times are represented by one random variable.

Taking these assumptions into consideration, the exemplary order picking system is transferred into a queueing model as shown in figure 9.

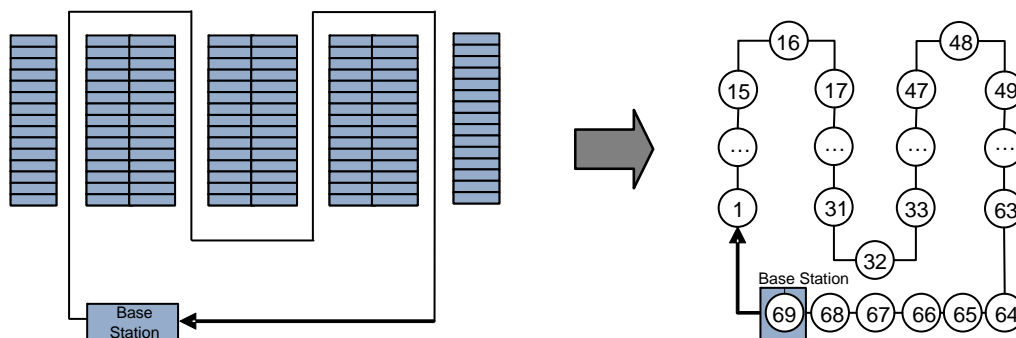


Figure 9: Exemplary order picking system as a queueing model

According to the presented approach each elementary space of the exemplary system is modeled as one queueing system with general service times. The elementary queueing systems 16, 32 and 48 represent the cross-aisles; queueing systems 64-68 represent the route from the last aisle to the base station. The base station is modeled as queueing system 69. The remaining queueing systems represent the elementary spaces within the aisles. The system capacities m_i are 1 each (no buffer space). Due to the routing strategy the visit ratio e_i is 1 for all queueing systems.

5.2. Throughput comparison: Simulation – Queueing theory – Static approach

To quantify the throughput reduction caused by blocking, the exemplary system is modeled using simulation. The activities and dynamic effects, which can be observed, are reflected in detail. We therefore assume that the results produced by simulation match the “real throughput of the exemplary system” best and consequently represent reference values for the subsequent analysis. The comparison of these reference values to the performance indicators as calculated by the queueing model shows that queueing theory is an adequate method to consider dynamic effects in manual order picking systems. The simulation results are also compared with the results of static approaches, which do not account for the blocking situations. This step will show to what extent static approaches overestimate real system throughput.

The sum of the dead and picking times is assumed to be $\text{LogN}(15,10)$ distributed. For queueing systems representing an elementary space within an aisle the service time has to include the case of picking as well as the case of passing the rack column without picking. Using the picking probability $p_i = 1/6$ (10 picks per order at 60 possible elementary spaces) the mean service time is calculated. The $\text{LogN}(15,10)$ distributed dead and picking time and a transit time of 1s result in a mean service time of 3,5s ($= 1/6 \cdot (15+1) + 5/6 \cdot (0+1)$). In order to determine the variability, we generate random numbers in a ratio of 1/6 (according to $\text{LogN}(15,10)$) to 5/6 (constant transit time of 1s with variance = 0) and derive the required statistical values. The base time is distributed according to $t_{\text{Basis}} \sim \text{LogN}(10,5)$. The service times of the cross-aisles are each 1s deterministic. The performance indicators of the queueing model are calculated using the method of Rall.

To calculate order throughput by means of static approaches we use the basic travel time model of Gudehus (Gud-05). With the assumptions of the exemplary system, the mean number of aisles x , in which the 10 picks are conducted, is calculated as 3,775. The mean path length per order $L(4 \text{ aisles})$ is 67,115m and with the given

speed a travel time of 67,115s is derived. Adding the total dead and picking times of 150s and the base time of 10s, results in an order throughput time of 227s which converts to a throughput of 15,85 orders per hour.

Assuming a linear relation between the number of customers in the system and the order throughput as calculated by the static approach, figure 10 shows the throughput trends of the three different methods of calculation used in this paper.

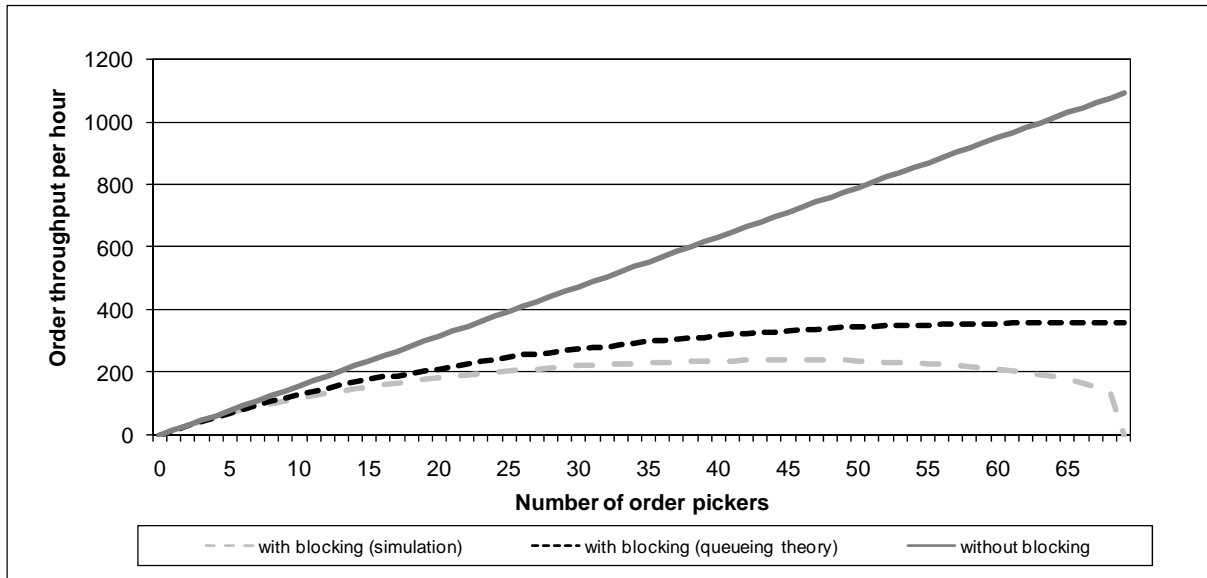


Figure 10: Throughput trends for different methods of calculations

Obviously throughput increases constantly when not considering blocking situations. The throughput trend as calculated by means of queueing theory includes the dynamic effects and draws near a saturation limit for rising number of order pickers. The throughput trend as calculated by simulation also shows the dynamic effects and furthermore reflects the intuitional guess that beyond a certain number of order pickers the blocking situations overcompensate the additional throughput of an extra order picker. The throughput continues to decrease until a deadlock situation occurs and the throughput is 0. In the center and right ranges of the above diagram, the difference between queueing theory and simulation results grows bigger. A reason for this can be found in the solution algorithms which are used by the framework of Rall (figure 8). Balsamo, de Nitto Personé and Onvural (Bal-01) note that for larger networks, errors in Akyildiz's algorithm (Aky-88) increase. An improvement of these algorithms must therefore be subject of future research activities. For practical purposes the center and right ranges in figure 10 can be ignored because real systems will mostly operate with a limited number of order pickers (\ll number of elementary spaces).

The overestimation of real throughput depending on the method of calculation is derived by using the following formulas:

$$\Delta = (X_{\text{Static Approach}} - X_{\text{Simulation}}) / X_{\text{Simulation}}$$

$$\Delta = (X_{\text{Queueing Model}} - X_{\text{Simulation}}) / X_{\text{Simulation}}$$

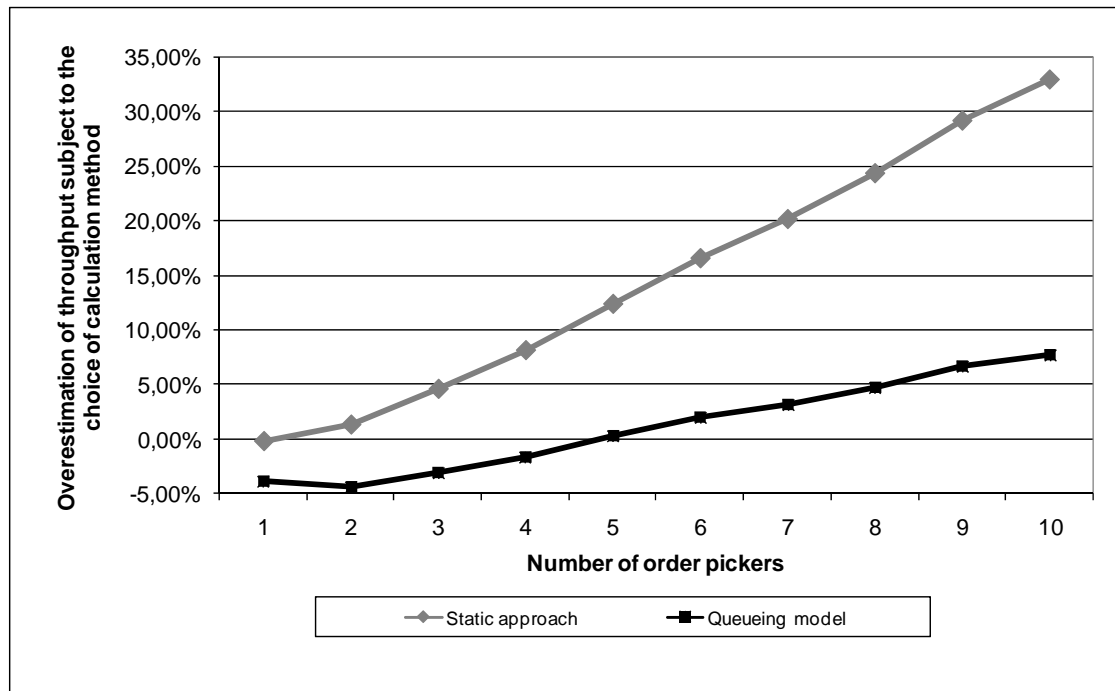


Figure 11: Overestimation of real throughput

Figure 11 shows the deviation between the results of simulation and the results of the static approach and the queueing model respectively. The results of the queueing model are within an acceptable range of $\pm 5\%$, if less than 9 order pickers work in the system (the maximum occupation rate of the elementary spaces thus is 12%). The application of static approaches without consideration of blocking however quickly leads to a noticeable overestimation of throughput.

In addition to the throughput performance, a system can be evaluated using cost criteria. For this purpose a productivity figure, for example “number of picks per man-hour”, can be calculated. From this we can derive the costs per pick. Figure 12 shows that in the case of static calculation methods, the productivity remains on a constant level. By contrast, the simulation as well as the queueing model demonstrates productivity losses.

When using static approaches in the rough planning phase of manual order picking systems, disregarding blocking situations can lead to a wrong estimation of system performance and thus to a false system design and cost estimates. The costs are underestimated because the individual picks are priced too low. However the results of the queueing model follow the real throughput trend and permit a more realistic estimation of expected system performance and costs respectively.

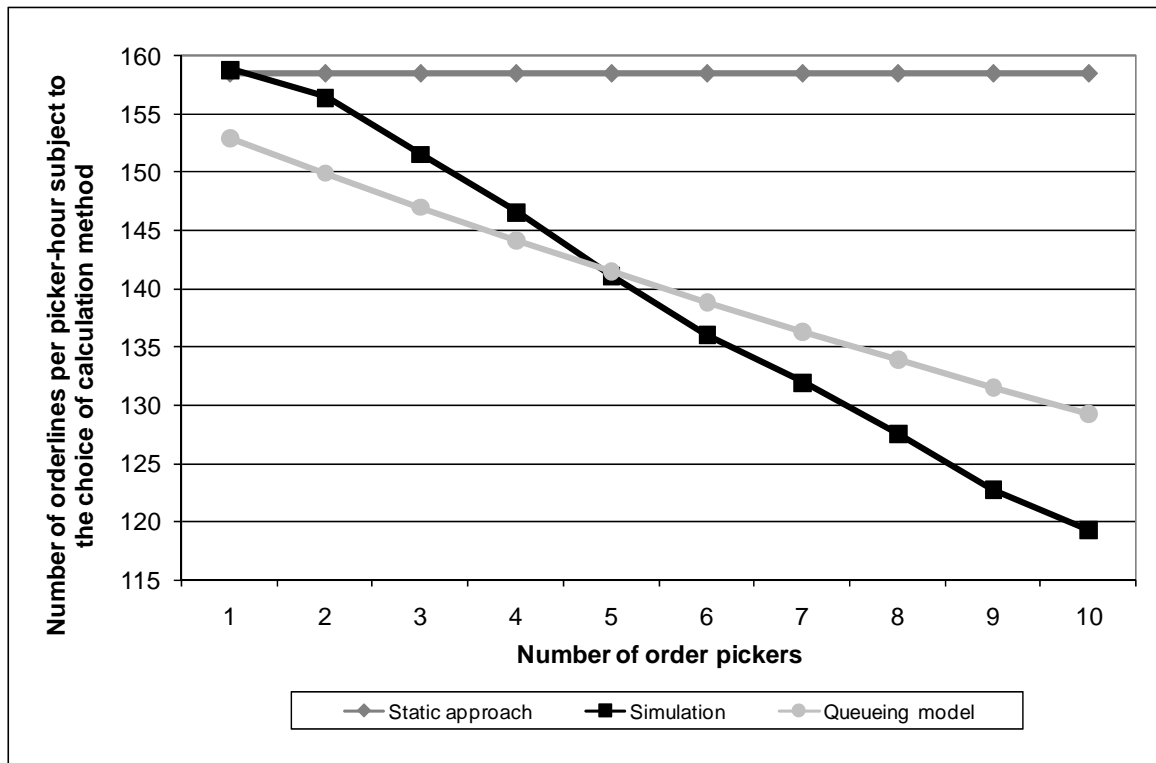


Figure 12: Productivity losses as a result of blocking

5.3. Throughput comparison: Simulation – Queueing theory

In the following we will compare the reference values of the simulation with the characteristic values of the queueing model in more detail for a practical range. In the first scenario the dead and picking time is assumed to be $t_{\text{Dead+Picking}} \sim \text{LogN}(15,10)$ distributed. The standard deviation of the base time is 5s. The mean base time varies between the values 10, 20 and 30s. We derive the service times and variabilities as shown in chapter 5.2 based on a picking probability $p_i = 1/6$. Again the method of Rall is used to compute the characteristic values. Figure 13 illustrates the percental deviation of the results within a practical range.

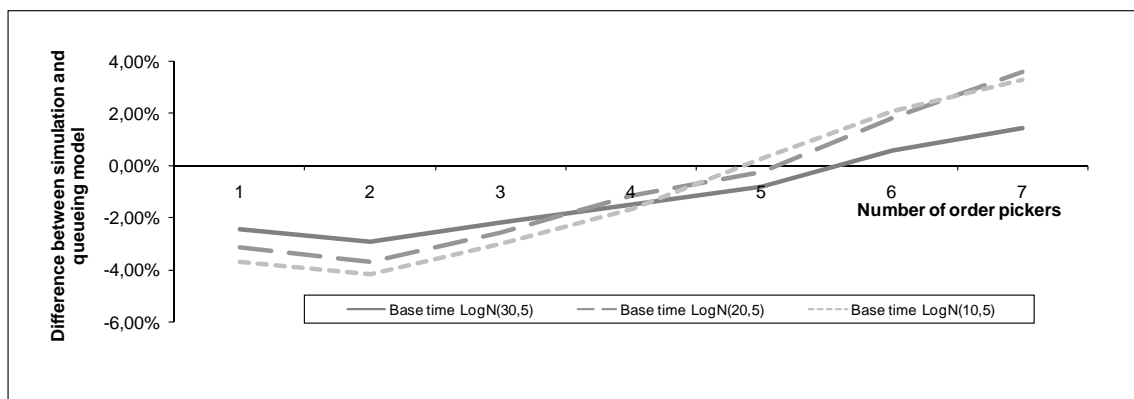


Figure 13: Percental deviation between simulation and queueing theory results with fix dead and picking time

At first, order throughput of the simulation is higher than analytical calculations. Starting at a certain number of order pickers, the queueing model delivers higher throughput values. For a mean base time of 30s, the relative deviation can be disregarded. This statement also holds for reduced mean base times of 10 and 20s respectively, if the number of order pickers is low. With this number rising, the deviation also rises and is approx. 8% for 10 order pickers.

The deviation increases if the base station does not represent a bottleneck and the variabilities at the elementary spaces rise (see figure 14). For a mean base time of 30s and a low number of order pickers the base station can already be identified as the bottleneck. The parameters of the base station then become the decisive throughput

driver. The base station becomes less of a bottleneck with decreasing mean base times. The queueing systems within the aisles then act as the throughput driver and their variabilities have an impact on the results of the queueing model. The deviation then increases with rising variabilities at the elementary spaces. Because the picking probability is 1/6 the order pickers will mostly pass the elementary spaces (service time = 1s). With $t_{\text{Dead+Picking}} \sim \text{LogN}(15,10)$ this results in an overall variability of 3,8837.

In a second scenario the standard deviation of the dead and picking times are varied and the influences analyzed. The queueing systems representing elementary spaces within the aisles have a mean service time of 15s. The associated standard deviation varies between the values 5, 20 and 25s. The base time is assumed to be $\text{LogN}(10,5)$ -distributed. The differences of throughput values calculated by queueing theory and simulation increase for high variabilities (see figure 14). For large standard deviations the service process at the rack columns have very high variabilities of 7,35 ($t_{\text{Dead+Picking}} \sim \text{LogN}(15,20)$) and 10,57 ($t_{\text{Dead+Picking}} \sim \text{LogN}(15,25)$) respectively. For the case of $\text{LogN}(15;7,5)$ -distributed dead and picking times the variability is comparatively low. The differences in the practical range lie between +/- 5%.

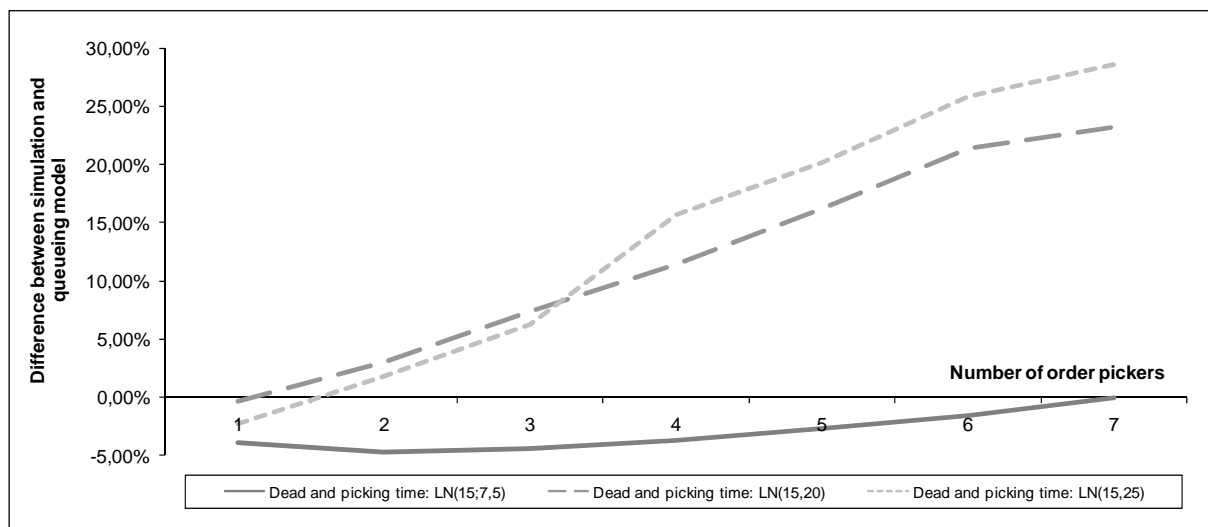


Figure 14: Percental differences between simulation and analytical calculation (fix base time)

The scenarios studied show that characteristic values derived analytically are relatively consistent compared to the results of simulation for low variability of the dead and picking times. The examples assume a base time which makes up approx. 5-12% of order throughput time. Gudehus (Gud-05) numbers the share of base time at approx. 10%. Thus the conformity with examples from the literature is ensured. For variabilities $c_{\text{Dead+Picking}}^2 > 1$ the differences between simulation and queueing model increase. In the range of three or less order pickers the error is below 10%. Another subject for future research is a more detailed analysis of the implications of high variabilities and possibilities on how to better take these implications into account in existing solution algorithms.

Compared to simulation queueing models offer a quick modeling and calculation of results. The concepts for modeling manual order picking systems using queueing models presented in this paper can easily be applied to numerous systems. The presented solution algorithms facilitate a quick calculation of characteristic values.

6. Summary and outlook

In order picking systems, the biggest cost factor of intralogistics, productivity losses can occur even for a small number of order pickers when they block each other within aisles or at the base station. In the course of the research project “Blocking in manual order picking systems with Picker-to-Part movements”, funded by the Deutsche Forschungsgemeinschaft (DFG), theoretical foundations to specify and quantify blocking phenomena will be developed.

To analyze systems that are subject to stochastic effects, queueing models were identified as a suitable tool. Using the presented modeling approach manual order picking systems – consisting of the main components base station, aisle and cross-aisle – can be transferred into a closed queueing network. Specific characteristics, such as the routing strategy can be considered in the queueing model by means of different parameters. The method of

Rall can be used to calculate characteristic values for closed queueing networks with finite buffer space and general distributed service times. The method satisfies the requirements that result from the modeling approach.

The following findings from simulation and queueing model shall be highlighted:

- In manual order picking systems blocking situation will lead to throughput losses, even for a small number of order pickers.
- The productivity of an order picker, measured with the number of picks per man-hour, decreases noticeably with each additional order picker.
- In contrast to static approaches, simulation as well as queueing theory is able to quantify the throughput losses caused by blocking situations.
- Queueing theory is a capable calculation method, because compared to simulation, results can be derived in short time and with comparable quality.
- The relative deviation between simulation and analytical calculation is below 5% for low variabilities, but increases with rising variabilities.

References

- [Arn-03] Arnold, D.: Abschlussbericht Modellierung und Optimierung von mehrstufigen Kommissioniersystemen (Forschungsvorhaben AR 160/14-1 der Deutschen Forschungsgemeinschaft), 2003.
- [Aky-87] Akyildiz, I.F.: General Closed Queueing Networks with Blocking. In: Proceedings of the 12th IFIP WG 7.3 International Symposium on Computer Performance Modelling, Measurement and Evaluation. Amsterdam: North-Holland 1987.
- [Aky-88] Akyildiz, I.F.: On the Exact and Approximate Throughput Analysis of Closed Queueing Networks with Blocking. *IEEE Transactions on Software Engineering* 14(1), 1988, S. 62-70.
- [Bal-01] Balsamo, S., de Nitto Personé, V. und Onvural, R.: Analysis of queueing networks with blocking. Norwell: Kluwer Academic Publishers 2001.
- [Bol-98] Bolch, G., Greiner, S., de Meer, H., Trivedi, K.S.: Queueing networks and Markov chains: modeling and performance evaluation with computer science applications. New York: John Wiley & Sons Inc. 1998.
- [Bou-96] Bouhchouch, A., Frein, Y. und Dallery, Y.: Performance Evaluation of closed tandem queueing networks with finite buffers. *Performance Evaluation* 26, 1996, S. 115-132
- [DCRM-08] Distribution Center Reference Model (DCRM): Ergebnisse der fortlaufenden Studie „Warehouse Excellence“, Institut für Fördertechnik und Logistiksysteme, Universität Karlsruhe (TH), 2008.
- [Fai-02] Faißt, B., Lippolt, C. : Staueffekte in Materialflusssystemen. F+H Fördern und Heben 52(8), 2002, S.506-508.
- [Gud-05] Gudehus, T.: Logistik (Grundlagen – Strategien – Anwendungen). Berlin: Springer 2005.
- [Gue-05] Gue, K.R., Meller, R.D. und Skufca, J.D.: The effects of pick density on order picking areas with narrow aisles. *IEE Transactions* 38, 2005, S. 859-868.
- [Hall-93] Hall, R.W.: Distance approximations for routing manual pickers in a warehouse. *IIE Transactions* 25(4), 1993, S. 76-87.
- [Kun-75] Kunder, R. und Gudehus, T.: Mittlere Wegzeiten beim eindimensionalen Kommissionieren. *Zeitschrift für Operations Research* 19, 1975, S. B53-B72.
- [Kre-08] Kregel, M., Turek, K.: Modellierung der Dynamik manueller Operationen in logistischen Systemen. 4. *Fachkolloquium der Wissenschaftlichen Gesellschaft für Technische Logistik*, Dresden, 2008.
- [Lün-05] Lünig, R.: Beitrag zur optimierten Gestaltung des Durchsatzes in Kommissioniersysteme für Stückgüter. Göttingen: Cuvillier 2005, zugl. Dissertation Technische Universität Ilmenau.
- [Mar-80] Marie, R. : Calculating Equilibrium Probabilities for $\lambda(n)/C_k/1/N$ Queues. *ACM Computing Surveys* 9(2), S. 117-125.
- [Rall-98] Rall, B.: Analyse und Dimensionierung von Materialflusssystemen mittels geschlossener Warteschlangennetze. Dissertation. Wissenschaftliche Berichte des Institutes für Fördertechnik und Logistiksysteme der Universität Karlsruhe (TH), Band 46, 1998.
- [Rat-83] Ratliff, H.D. und Rosenthal, A.S.: Order-picking in a rectangular warehouse: A solvable case of the travelling salesman problem. *Operations Research* 31(3), 1983, S. 507-521.
- [Rood-01] Roodbergen, K.J. und de Koster, R.: Routing order pickers in a warehouse with a middle aisle. *European Journal of Operations Research* 133, 2001, S. 32-43.
- [Rub-99] Ruben, R.A. und Jacobs, F.R.: Batch construction heuristics and storage assignment strategies for walk/ride and pick systems. *Management Science* 45(4), 1999, S. 575-596.

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