

# Mathematical methods for the configuration of transportation systems with a focus on continuous and modular matrix conveyors

Mathematische Methoden zur Konfigurierung von Materialflusssystemen mit Fokus auf Stetigförderer und modulare Fördermatrizen

*Nikita Shchekutin  
Sergei Zobnin  
Ludger Overmeyer  
Vyacheslav Shkodyrev*

*Institute of Transport and Automation Technology  
Leibniz Universität Hannover*

*Control Systems and Technologies Department  
Peter the Great St. Petersburg Polytechnic University*

**T**he lack of flexibility in logistic systems currently on the market leads to the development of new innovative transportation systems. In order to find the optimal configuration of such a system depending on the current goal functions, for example minimization of transport times and maximization of the throughput, various mathematical methods of multi-criteria optimization are applicable. In this work, the concept of a complex transportation system is presented. Furthermore, the question of finding the optimal configuration of such a system through mathematical methods of optimization is considered.

*[Keywords: logistics, material flow system, matrix conveyor, warehousing]*

**D**er Mangel an Flexibilität bei heutigen Logistiksystemen führt zur Entwicklung von neuartigen Fördersystemen. Um die optimale Konfiguration der Systeme anhand aktueller Zielfunktionen, wie beispielsweise Minimierung von Transportzeiten und Maximierung des Durchsatzes, herauszufinden, sind verschiedene mathematische Ansätze zur simultanen Optimierung verschiedener Kriterien anwendbar. In dieser Arbeit wird das Konzept für ein komplexes Materialflusssystem vorgestellt. Außerdem wird die Frage nach einer Konfigurierung solcher Systeme durch mathematische Optimierungsmethoden betrachtet.

*[Schlüsselwörter: Logistik, Materialflusssysteme, Fördermatrix, Intralogistik]*

## 1 INTRODUCTION

Warehousing offers an enormous potential for optimization in all industries. Due to the shortening of product life cycles, speeding up of marketing, and the complicity of requirements for material handling systems from the mechanical point of view, warehousing systems need to become more flexible [Fur10]. From the other point of view, today's material systems are mechanically predefined – functionalities such as sorting or buffering are locally bound into the system. Thus, new solutions must provide flexibility for warehousing systems from both the control and mechanical point of view. [Jod06]

These fast changing requirements lead to the demand for a reconfiguration of the system, taking into account the lack of time. Therefore mathematical approaches of configuring the system must be considered.

This article is built as follows: firstly, a new kind of continuous conveyor is presented. It consists of small-scale, multi-directional transport modules. Then, the mathematical description of the problem which defines the required parameters and optimization criteria is set. Afterwards, the mathematical optimization methods to solve this problem are proposed and analyzed.

## 2 RELATED RESEARCH

### 2.1 ISSUES IN WAREHOUSING

Two main issues of material handling systems can be identified. The first one is inflexibility - it is of high costs and efforts to change the configuration of a system once it is designed and installed. However, due to product indi-

visualization, reconfiguration of material flow systems is a necessary part of nowadays industry to make needed materials available in the right time, at the right place, and in the right quantity.

Furmans et. al described in [Fur10] that material flow systems should be modularized to provide enough adaptability through the assembly of single elements containing a plug-and-work ability. It is also furthered on control principles – due to the high number of elements, it is not possible to program a control for each single element individually. The solution for this case is a decentralized control with the identical control logic as if each element used its own program running without central supervision. For instance, the FlexConveyor platform was developed and it is based on multiple, identical square modules that can easily be plugged and unplugged, where one module is at least the size of one packet. [May09]

In comparison to the previously described project, the specialty of the small-scale conveyor modules developed at the Institute of Transport and Automation Technology consists of the fact that the modules are much smaller than the transported materials (Fig. 1). The resulting material flow system is built up from the modular matrix in combination with continuous conveyors. Another special point in this system is the fact that the goods are transported simultaneously to its data packages. [Shc15]

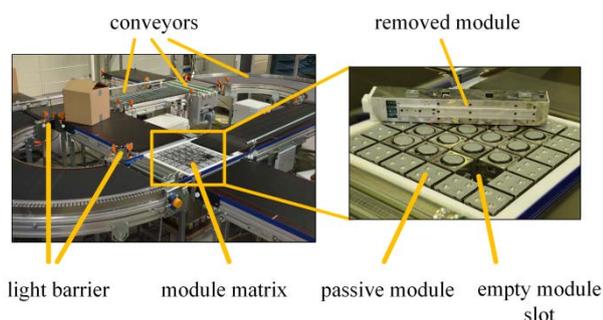


Figure 1. Figure 1. Overview of the transportation system developed at the ITA [Kri15]

The second main issue in warehousing is the search of the optimal configuration of a system according to the constantly changing production requirements. The modularity of the system makes layouts easily adaptable. However, reconfiguration planning still requires some costs and efforts and was rarely addressed in the related research. The product individualization leads to the fact that calculation of the optimal configuration of a material flow system must occur automatically and also simultaneously to the changing requirements.

Therefore, algorithms for an automated calculation of a system's optimal configuration are necessary. First of all, however, the basic principles of warehousing system configurations must be reviewed.

## 2.2 SPATIAL ARRANGEMENT OF WAREHOUSING SYSTEMS

When speaking about the rationalization of a material flow system, one question always comes up – what makes more sense? To optimize single processes within the framework of an existing layout or change the structure of the whole system? [Hei79]

To avoid answering this question during the exploitation of a material flow system, it must be decided in advance what is the main motivation of planning and building a material flow system?

First of all, an extensive material flow system planning must justify long-term investment in warehousing in terms of technical, economical as well as organizational aspects. Main requirements to future planning were formalized in [Hei97]: it must be systematical, methodical, dynamical, iterative, flexible, adaptable, exact, complete, explicit, continuous, and economical.

The important fact is that the classical factory planning deals with “relatively sure information” – this information normally comes from planning or request prognoses. If the information is not sure, then processing it belongs to the process control [Hei11].

Influencing factors on system planning can be divided into internal and external factors. External factors will not be considered in this work: laws, regulations, norms, and values as financing, requirements from customers and suppliers as well as influencing factors of technologies, as for instance, new processes and machines. Also such internal factors as strategy of industry management, border conditions, checkpoints and restrictions inside of the company as for example dimensions of space or huge machine fundamentals will only be partially considered in this work. [Hei97]

Fig. 2 shows a basic concept of planning a material flow system. It can be divided into the seven steps; methods and recommendations for each step are presented in the neighboring boxes. Steps 3, 5 and 6 are in focus of our research.

According to fig. 2, one of the main issues is creating an appropriate mathematical model, as models are only conditionally practically implementable because they do not consider qualitative influencing factors properly. Therefore, our task is firstly to find an appropriate model, which shows the system correctly, even though it is too complex.

Methods	Statistic	Time study	Cybernetic Models	Catalogues
	Description in technical terminology	Collection and processing of data	Brainstorming	Brainstorming
Step	1 Task assignment	2 Planning data analysis	3 Design of process variants	4 Design of technical equipment
	5 Dimensioning, checking and evaluation of variants	6 Detailed planning	7 Realization	
Methods	Prognosis calculation	Graphical representation	Variants development	Variants development
	Graphical representation	Graphical representation	Graphical representation	Graphical representation
Methods	Mat. Models	Planning tables	Planning tables	Planning tables
	Simulation and experiments	Project management	Project management	Project management
Methods	Efficiency analysis	Critical path analysis	Critical path analysis	Critical path analysis
	Cost-benefit analysis	Decision trees procedure	Decision trees procedure	Decision trees procedure
Methods	Static and dynamic capital budgeting			

Figure 2. Sequence of planning a material flow system and exemplary methods and helping tools of single planning steps [Jün89]

### 3 MATHEMATICAL DESCRIPTION OF THE SYSTEM

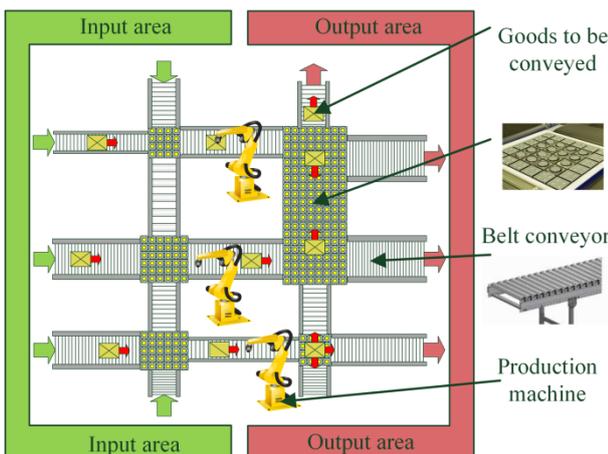


Figure 3. Concept of an intelligent production system

An example of a possible configuration of an intelligent production system is presented in figure 3. This system consists of multiple input and output ports, production machines, continuous conveyors and modular matrices,

built up from transport modules. Each packet enters the system with its own data package, which contains the information about its size, required production steps and destination. Based on this information the system reserves an individual route and transfers the packet according to this route; conveyor elements organize themselves individually for this purpose and because of the route reservation conflict situations such as deadlocks are avoided.

Before starting the mathematical description of the system some principle assumptions have to be settled: input and output ports are situated outside the system and the model of entering packets into the system is out of the scope of our work; belt conveyors are able to transport goods in both directions; all conveyor modules are of square shape; the setting of data packages and reading its information is also out of scope of our work.

Then let  $Inp_i$  ( $i = 1 \dots I$ ) and  $Outp_j$  ( $j = 1 \dots J$ ) denote a set of inputs and outputs of the transportation system. Let  $MoP_k$  ( $k = 1 \dots K$ ) denote a set of production operations. Let  $M_p$  ( $p = 1 \dots P$ ) denote a conveyor module. Let  $C_q$  ( $q = 1 \dots Q$ ) be a belt conveyor piece. Let  $Position: f(o) \rightarrow Z^2$  be a function that returns a tuple of object coordinates in a two-dimensional discrete warehouse space. The transportation system can then be represented by the following tuple:

$$\left( \begin{array}{l} \{Inp_i, Position(Inp_i)\}, \{Outp_j, Position(Outp_j)\}, \\ \{MoP_k, Position(MoP_k)\}, \{M_p, Position(M_p)\}, \\ \{C_q, Position(C_q)\} \end{array} \right).$$

Based on this description we consider two optimization problems and their corresponding mathematical models in the domain of continuous conveyors and modular matrix conveyors.

The first problem is the “weak problem” of optimization; it stands for a task of optimizing the configuration of the system assuming that the system topology is already defined. In this problem, we consider a restrictive environment in which only linear conveyors exist.

The second problem is the “strong problem” of optimization, it refers to an extended version of the first optimization problem, when a system topology itself is not defined and all conveyor shapes and modifications are available. This problem will not be considered in this article and will be looked into in further research.

## 4 THE WEAK PROBLEM OF TRANSPORTATION SYSTEM OPTIMIZATION: PROBLEM AND ALGORITHMS

### 4.1 DEFINITION OF PROBLEM

Using the notations defined in the previous paragraph and some additional assumptions, we formalize the optimization problem as follows.

Let *ConveyorLength*:  $g(C_q) \rightarrow Z^1$  be a function that returns the length of an independent linear conveyor. Let  $MAT_b$  ( $b = 1 \dots B$ ) denote a set of modular matrices. Both  $\{C_q\}$  and  $\{MAT_b\}$  are generated during optimization to form a possible solution of the problem.

We aim at minimizing the resulting number of belt conveyors  $\{C_q\}$ , number of conveyor modules  $\{M_p\}$ , number of modular matrices  $\{MAT_b\}$  and total linear conveyor length  $\sum_q ConveyorLength(C_q)$ . We also consider the following key performance indicators (KPIs) generated in a simulation environment for each of the considered solutions:

- Travel time indicator *TT*
- Travel distance *TD*
- Conflicts *Col*
- Idle time *IT*
- Energy consumption *EC*

The KPIs form a vector:

$$SimKPI = (TT, TD, Col, IT, EC)^T$$

The resulting multi-criteria optimization problem is defined as follows:

$$\alpha * \{C_q\} + \beta * \{M_p\} + \gamma * \sum_q ConveyorLength(C_q) + \delta * \{MAT_b\} + \omega * SimKPI \rightarrow \min,$$

where  $\alpha, \beta, \gamma, \delta$  are coefficients and  $\omega$  is a coefficient vector for KPIs.

### 4.2 ALGORITHMS

The defined optimization task belongs to a combinatorial one. The exact algorithms are designed for a particular problem taking into account its properties and are usually applicable to small-scale engineering problems which is not appropriate for us. That is why in our case local search strategies will be used, when through moving

from a solution to another one in its neighbourhood according to some pre-defined rules, minimization of a function on a finite set of points is achieved. [Pir96]

In order to solve large problems as ours, metaheuristic approaches are used. Such methods are not bound to any particular problem or domain and provide good solutions in a relatively short time. [Xio15]

The most appropriate class of metaheuristic-based methods for solving the optimization problem defined above are “trajectory-based optimization methods”. They are inspired by processes and concepts, which have nothing in common with optimization. For instance, the Simulated Annealing method was developed on the basis of the physical process used in metallurgy. [Aze92]

Approaches of this type explore the problem space via transition from one feasible solution (a possible vector without optimal parameter bindings) to another. The transition procedure is controlled by algorithm-specific rules but the general layout of trajectory-based algorithms is the same.

The majority of these techniques starts with an initial feasible solution to the problem (far from optimal, usually created randomly); then a new solution is created. If the next generated solution has a better objective value than the current one it becomes the current solution and the search moves on to the next iteration.

Comparing other methods, which are based on the same principles (Tabu Search and Genetic Algorithms), the method of Simulated Annealing has one advantage: it is easier to implement when addressing such general tasks with a lot of assumptions, as ours. Advancing further into the area of subproblems, other methods can be applied. In next subchapter we describe briefly the implementation of the Simulated Annealing method in application to the search for the optimal configuration of a material flow system.

### 4.3 SIMULATED ANNEALING METHOD

The method is based on a random drawing of new solutions, which are accepted if either they improve the system or according to some probability. At the start, almost all solutions are accepted in order to explore the solution space; with time the system becomes more and more selective in accepting new solutions. In the end, only configuration changes that really improve our goal function are accepted. Choosing the so-called “cooling schedule”, which determines the probability of acceptance of a new solution, is a special topic to discuss, but normally a Boltzmann-like distribution based on the analogue of thermodynamics is used.

During such transitions additional techniques are used to avoid sticking into local optima. These techniques

are particularly recommended when there is limited time for search.

In order to apply the Simulated Annealing to a weak problem, a clear definition of a feasible solution is required; this definition is subject to later programmatic encoding. A solution vector in our case contains the following information: location of production machines, location and size of clusters, location of belt conveyors.

Two operations in the feasible solution area need to be defined: the generation of an initial solution and the transformation to a neighborhood solution with slightly different parameters, with a critical decision for such strategy.

The general algorithm of an initial solution generation consists of the following steps and is conceptually presented in the figure 4:

1. Assumption that all inputs and outputs are directly connected with belt conveyors (line 1, here and further at the fig. 4)
2. Production machines are placed randomly in the system (lines 5-6 )
3. Some parts of the belt conveyors are removed from the system (lines 2-4)
4. Some modules are placed randomly, possibly forming matrices (lines 7-8)

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**Algorithm 1: Initial solution generation**

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**Input:** *inputs, outputs, productionOperations, systemBoundaries,  $\alpha, \beta$*

```

1 conveyorPieces =
  generateInitialLayout(systemBoundaries,
    inputs, outputs)
2 for c in conveyorPieces do
3   if random() <  $\alpha$  then
4     remove(c, conveyorPieces)
5 for po in productionOperations do
6   assignPosition(po)
7 for i = 0; i <  $\beta$ ; i++ do
8   m = placeSeedModule(randomAppropriatePlace());
   growMatrix(m)
    
```

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Figure 4. A concept of an initial solution generation.

The general algorithm of the transformation to a neighboring solution consists of the following steps:

1. Each of the production machines and clusters is slightly moved in a random direction with some probability (line 1-3 and lines 4-8, here and further at the fig. 5)
2. A number of clusters and belt conveyors is added to random places in the system (lines 9-10, 14-15)
3. A number of existing clusters and belt conveyors is removed from the system (lines 5-6, 11-13)

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**Algorithm 2: Transformation to neighbouring solution**

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**Input:** *productionOperations, conveyorPieces, matrices,  $\gamma, \delta, \zeta, \epsilon, \theta$*

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1 for po in productionOperations do
2   if random() <  $\gamma$  then
3     movePONearby(po)
4 for m in matrices do
5   if random() <  $\delta$  then
6     remove(m)
7   else
8     moveMatrixNearby(m)
9 for i = 0; i <  $\zeta$ ; i++ do
10  m = placeSeedModule(randomAppropriatePlace());
   growMatrix(m)
11 for c in conveyorPieces do
12  if random() <  $\epsilon$  then
13    remove(c, conveyorPieces)
14 for i = 0; i <  $\theta$ ; i++ do
15  addConveyorPiece(randomAppropriatePlace());
    
```

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Figure 5. Concept of the transformation to neighboring solution

There are 2 basic methods for an algorithm stopping rule: the search stops accepting the latest optimal solution, if the goal function was not improved for a defined number of steps or if the number of the accepted moves is less than the number of consecutive steps for the latest acceptance probability.

As mentioned before, it is a general concept of solving our problem and a detailed implementation is planned for further research. For this purpose, first of all an integration into the existing simulation environment has to be considered.

## 5 CONCLUSION

The problem of finding the optimal configuration of a material flow system was addressed in this work. Taking into account the state of the art in a spatial arrangement in warehousing, a concept of an optimization of a material flow system configuration was proposed and divided into strong and weak problems. In order to evaluate the proposed concept, it was applied to the developed material flow system. For this purpose, a mathematical description and an optimization function were defined. The existing optimization methods were analyzed and as the first approach a Simulated Annealing was chosen as an appropriate one. Further research is planned in the following areas: the implementation of a chosen optimization method as well as the considering of a strong problem of optimization in application to a configuration of the material flow system.

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**Nikita Shchekutin, M.Sc.**, Research Assistant at the Institute of Transport and Automation Technology, Leibniz Universität Hannover.

Nikita Shchekutin was born 1989 in Togliatty, Russia. Between 2006 and 2012 he studied Information technologies at the Peter the Great St. Petersburg Polytechnic University.

Address:

Institute für Transport- und Automatisierungstechnik,  
Leibniz Universität Hannover, An der Universität 2,  
Garbsen, Germany.

Phone: +49 (0)511 762 18-339,

Fax: +49 (0) 511 762 40 07,

E-Mail: [nikita.shchekutin@ita.uni-hannover.de](mailto:nikita.shchekutin@ita.uni-hannover.de)